Stability in Domain Wall Switching: Semianalytic rate via Mapping to 1D

P. B. Visscher & Ru Zhu

MINT Center and Department of Physics and Astronomy
The University of Alabama

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Background and summary

Domain wall motion and switching is important in several types of storage devices, including exchange-coupled and graded-anisotropy devices.

It is especially difficult to estimate the switching rate for spontaneous switching (H=0), to evaluate stability, because direct LLG simulation is impossible (real time ~ tens of years > time steps).

Rate has been computed for coherent switching of uniaxial particles using the Fokker-Planck equation [Kramers 1940, Brown 1963]. This works because the problem is 1D.

Random pinning

\[
\text{rate} = \frac{W}{\epsilon_{1/2}} \sim 0.2 \text{ to } 0.8
\]

where \(W\) = domain wall width/particle length, \(C\) = curvature of barrier (relative to Stoner-Wohlfarth)

Examples of Energy Landscape - energy vs. magnetization (\(M\)) curve

Stoner-Wohlfarth particle (limit as exchange \(A\) → infinity)
- Maximum slope \(dE(M)/dM\) = \(-\mu_0H\) is the coercivity
- Maximum height is the stability barrier

Anisotropy-graded system: \(K(x) \sim x^2\) switches by domain wall motion.

Calculating energy landscape for \(K(x) \sim x^2\) (minimum-energy path)

Stable, recoverable and independent of parameters here

Fast Sweep

Slow Sweep

Sweep \(m_{\text{initial}}\) (1) set \(m_{\text{initial}}\)

Adjust \(H_s\) to force this rate:

\[
\dot{H}_s = -C(m_s - m_{\text{desired}})
\]

LL algorithm finds minimum energy (at fixed \(M_s\))

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Deriving 1D equation of motion for \(M_s\)

We start from the Landau-Lifshitz equation:

\[
\frac{dM}{dt} = -\gamma M \times H - \alpha M \times (M \times H)
\]

Adding all the cells to get the rate of change of the total magnetic moment:

\[
\frac{dm_s}{dt} = \mu(m_s)(H_s - H_{\text{pin}}(m_s)) + m_{\text{random}}
\]

where \(\mu(m) = \alpha\gamma M_s \int \sin^2 \theta d^3r\)

is the domain wall mobility related to the diffusivity by the Einstein relation \(D = \mu kT\).

Using Kramer's method to calculate the Switching Rate

Fokker-Planck equation is

\[
\frac{d\rho(m)}{dt} = \frac{dJ}{dm} \text{ where } J = \rho\mu(H_s - H_{\text{pin}}(m_s)) - D d\rho / dm
\]

If the current is zero (steady state), this has the exact solution

\[
\rho(m_s) = e^{[ - E(m_s) / k_BT ]} \text{ where the prefactor is }
\]

\[
r_s = \int [ e^{[ - E(m_s) / k_BT ]} ] dm = \int [ e^{[ - E(m_s) / k_BT ]} ] dm = \frac{1}{C^{1/2}} \exp\left\{ -E_s / k_BT \right\} \text{ Approx}
\]

In the second integral, if the stability factor \(E_s / k_BT\) is large, only the \(E(m)\) near the maximum (see figure) will be important. If we approximate \(E(m)\) as a quadratic function (as shown in the inset) then this is a Gaussian integral, which is easy to integrate.

Final Result for Rate

Switching Rate

\[
\frac{W}{C^{1/2}} \alpha\gamma H_s \left( \frac{E_s}{\pi k_BT} \right)^{1/2} \exp\left\{ -E_s / k_BT \right\}
\]

Which is close to Brown’s result except for the \(1/2\) factor.